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On Pair Annihilation and the Einstein-Podolsky-Rosen Paradox

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Abstract

Discussion is given to the experimental facts that are associated with 'pair annihilation', as a real example, rather than a gedanken experiment, to illustrate the Einstein-Podolsky-Rosen paradox. It is shown how the paradox disappears in a nonlinear relativistically covariant spinor field theory of this author, which takes the *single interaction*, rather than *many free particles*, as the elementary entity. In this theory there is no actual annihilation of matter. Rather, the observed facts that are conventionally interpreted as 'pair annihilation' are *derived* from an exact solution of the nonlinear field equations for the interacting pair in a particular deeply bound state. This solution reveals the observed facts, including the energy separation of $2m$ from the asymptotic state where the particles can be assumed to be (almost) free, and the prediction of two distinguishable currents whose phases are correlated by a 90° difference and are polarized in a common plane that is perpendicular to the direction of propagation of interaction with a detecting apparatus. The paradox disappears essentially because of the rejection by this theory (in principle and in the exact mathematical formalism) of any physical description in terms of truly uncoupled partial systems.

1. *Introduction*

In a recent discussion of the development of contemporary physical theory, Dirac (1963) emphasized the fact that in its present state, physics suffers from two kinds of difficulty. The first (which was called 'Class One difficulty') is concerned with the logical consistency of the quantum theory. The second (called 'Class Two difficulty') is concerned with the mathematical consistency of the necessary extension of quantum mechanics to a relativistic quantum field theory, in order to describe high-energy physics. His comments on these two difficulties were as follows:

> 'I have disposed of the Class One difficulties by saying that they are really not so important, that if one can make progress with them one can count oneself lucky, and if one cannot, it is nothing to be genuinely disturbed about. The Class Two difficulties are the really

serious ones. They arise primarily from the fact that when we apply our quantum theory to fields in the way we have to if we are to make it agree with special relativity...we have equations that at first look all right. But when one tries to solve them, one finds that they do not have any solutions.'

A major effort of present-day research in theoretical physics is being devoted to investigations of possible resolutions to Dirac's Class Two difficulties. The current studies of axiomatic field theory† and the S-matrix approach (Chew, 1961), as well as Dirac's own recent studies of quantum field theory (Dirac, 1966), are representative of this effort.

In addition, and in contrast with the recent approaches which attempt to maintain the basic postulates of the quantum theory, this author has been investigating a relativistic theory that is based entirely on the continuous field concept and where quantization plays no role.‡ The aim is to study the outcome of an extension from the Faraday-Einstein conception of field theory and, in particular, to construct a general theory whose formalism is both demonstrably mathematically consistent and contains, in the proper limit (of sufficiently low-energy-momentum transfer within an interacting system) the *formal* features of nonrelativistic quantum mechanics.

These studies have revealed that, indeed, one can construct a (mathematically consistent) covariant and deterministic formalism that predicts, for example, the correct quantitative spectrum of the hydrogen atom—*including the Lamb shift.* In addition, an *exact solution* of the coupled field equations, for a particle–antiparticle pair, has been found that relates to all of the experimental observations that are conventionally interpreted in terms of 'pair annihilation'. The latter observations can be associated, in turn, with an *actual experiment* of the type that is discussed by Einstein *et al.* (1935) in their argumentation against the logical consistency of the quantum theory.§

Dirac's comments about the Class One difficulties could be interpreted to mean that the argumentation which challenges the logical consistency of the Copenhagen interpretation of the quantum theory is unimportant, so long as quantitative predictions can be made in a

† See, for example, *Axiomatic Field Theory*, Vol. I, ed. by Chretien and Deser. Gordon and Breach, New York and London (1966).

‡ The philosophical aspects of this approach are discussed in Sachs, M. (1964), *British Journal of the Philosophy of Science*, **15**, 213; *Synthese*, **17**, 29 (1967).

§ The latter will be referred to hereafter as EPR.

mathematically consistent way. Still, any argumentation which relates to the logical consistency of the approach and also proposes a bona fide experiment to check the validity of its contentions, must be taken into account. Thus, the main purpose of this article is to demonstrate how a deterministic field theory whose mathematical structure is different and is interpreted differently from the conventional theories, can resolve not only the Class Two difficulty of Dirac's discussion, but at the same time can also remove the paradox of EPR argumentation (a prime example of a Class One difficulty in present-day physics). The uniqueness of this method to resolve both of these difficulties at once is not claimed. It is only presented to demonstrate one example of a different approach to high-energy physics that can accomplish such a resolution. The implication that is intended, however, is the idea that the problem of mathematical inconsistency may not be resolvable without removing the problem of logical inconsistency at the same time.

In the following section, the argumentation of Einstein, Podolsky and Rosen, as well as some of the counter-arguments and attempted resolutions, will be briefly surveyed, including the approach that is discussed in this paper. In Section 3, the experimental facts about a crucial test (of the type discussed in the EPR paper) will be outlined and related to the predictions of this author's approach. It will be demonstrated there how the experimental facts about pair annihilation, which seem to corroborate the quantum theory, agree precisely with the quantitative predictions of the considered deterministic field theory. It will be shown how the removal of the EPR paradox results here from the features of a mathematical formalism that are necessitated by the interpretation of the fundamental theory in terms of the elementarity of the interaction (rather than the elementarity of the particle) and the requirement for a *complete* description of the interaction in terms of the underlying field variables.

2. *The Einstein-Podolsky-Rosen Paradox*

In their historic paper (1935), Einstein, Podolsky and Rosen analysed a gedanken experiment in which one measures the dynamical variables of one part (A) of an uncoupled two-particle system (AB) by making measurements on B, which was previously bound to A and has since been separated by a mechanism that does not effect the correlation of the wave functions of the partial systems. They thereby demonstrated that an experimental situation could be created in which one can determine, to arbitrary accuracy, the dynamical variables of a microscopic entity A (or B) by measuring the properties of B (or A)—without having the measuring apparatus disturb the part A (or B) of the system in any way. Thus, they concluded that the dynamical variables of this microscopic system are 'predetermined', implying that it must have a complete description. On the other hand, the corresponding quantum mechanical solution for this partial system does not contain complete information about A (or B) and therefore it cannot represent the maximum attainable knowledge about this microscopic entity. Their conclusion, therefore, was that *any assertion about the completeness of the quantum mechanical wave function description of the system is paradoxical.*

Bohr (1949) ruled out the EPR paradox by rejecting their initial *implicit assumption* which asserts the existence of a one-to-one correspondence between the individual partial system and a precise mathematical description—even though the two partial systems can be assumed to be completely separated. Consequently, Bohr and Einstein were each led to different logical conclusions about the essential features of 'measurement' because they each started from a different axiomatic base in regard to the underlying epistemology—Einstein insisting on intrinsic determinism and Bohr on intrinsic nondeterminism—to describe the two- (or more) particle system.

If one should insist on the implicit assumption in the EPR argument, then a resolution of the paradox might be attained by interpreting the quantum mechanical wave function as relating to a 'classical' statistical average over an ensemble of individual systems. Einstein himself took this view. If, *in addition*, one should also insist on the correctness, *in principle*, of the linear eigenvalue formalism of the quantum theory, then 'hidden variables' would have to be introduced into the formalism in order to complete the description. Well-known studies along these lines were undertaken by Bohm (1952). Along with Bohm's approach, however, the possibility must also be considered that the linear eigenvalue structure of the *formalism* of the quantum theory may not be valid, *in principle*, even though its mathematical form is (empirically) correct in a proper limit. It is the latter possibility, *without the introduction of 'hidden variables'*, that will be discussed in this article.

Shortly after the EPR paper appeared, Furry (1936) proposed that the quantum picture could be left intact and the EPR paradox removed, if there would be a contribution to the Hamiltonian for the two-particle system (that would be effective only when the partial systems are sufficiently weakly coupled) that could break the correlation which existed previously when they were bound—thereby changing the description to a mixture of (uncorrelated) states. A crucial test of this hypothesis was the experiment of Wu & Shaknov (1950) on the correlation of the spins of the photons that are created when a particle–antiparticle pair annihilates. Such an experiment is precisely the type that is discussed in the EPR paper. The correlation of the 'noninteracting' systems relates here to the two photons that are supposedly created when an electron–positron pair annihilates in the singlet S-state.

The latter investigators measured, as a function of scattering angle, the coincident counts for radiation that was originally produced in pair annihilation and subsequently scattered by charged particles (by the Compton effect). The ratio of coincident counts, as a function of scattering angle, is sensitive to the correlation of the polarization orientations of the photons. A total correlation or lack of correlation would imply different theoretical ratios. The Wu-Shaknov experiment confirmed, to within the experimental accuracy, the correlation that is implied by the quantum theory. Thus, the long-range modification of the quantum mechanical Hamiltonian that was suggested by Furry did not agree with the experimental observations and thus did not provide a resolution for the EPR paradox.

This author has been investigating a general theory which has a formalism that differs entirely from that of the quantum theory when the former is expressed in its exact form—even though it approaches the standard quantum mechanical equations in the low-energy limit. The approach is based entirely on the classical field concept and it is expressed in terms of coupled *nonlinear* 'classical' spinor field equations, with all coupled fields mapped onto *a single space–time continuum*. When one considers the coupling of one 'particle' to a measuring 'apparatus', then the field equations for the two separated partial systems approach a linearized form. On the other hand, the EPR paradox concerns more than one particle coupled to a measuring apparatus. In particular, when one considers the relatively strong Coulomb coupling of the particle and antiparticle that come into close enough proximity to yield the effect that is conventionally interpreted as 'annihilation', the portion of the field equations that relates to these two interacting particles *must remain nonlinear* (within this approach)—even though the electron–positron pair, *as a unit*, approaches an uncoupling from the measuring apparatus. Thus, such an approach cannot consider the combined positron and electron in terms of a linear superposition of states.

An important question that arises at this point is the following: How can a deterministic, single field approach explain the experimental facts that are usually interpreted in terms of *separate* photons that are produced when *separate* particles annihilate each other *at arbitrary times*? To answer this question within the present theory, *a solution* of the underlying field equations must be shown *to predict* the experimental facts. Indeed, one of the striking successes of the 'classical' relativistic field theory under study has been such a derivation. In the following section, the detailed correlation between the mathematical solution that was obtained and the experimental facts that relate to pair annihilation will be analyzed.

Before commencing, it is important to point out that within the framework of this theory, the *photon* does not exist as a bona fide interacting particle. This introduces no difficulty in explaining most experiments that are supposed to involve photons (e.g. the photoelectric effect, the Compton effect) since the latter can be equally explained in terms of charged matter that is interacting over large distances. Wheeler & Feynman (1945) were among the first to discuss this in the literature. Of course, one may, for convenience, refer to the asymptotic solutions of Maxwell's equations that describe distant sources as 'photons'. Still, there are cases in which it is important to keep in mind the distinction between the latter and the strict definition of the 'photon' as a quantum of the source-free Maxwell radiation field. In particular, the 'annihilation process' is an experimental observation that is conventionally interpreted in terms of the presence of photons in the universe at times when there is no charged matter in existence.† It will be shown below that the experimental facts which pertain to pair annihilation can be explained in terms of *a bound state* of a particle–antiparticle pair. Thus, the photon does not play any role, within this approach, and the results of experiments that are usually interpreted in terms of the annihilation of matter must follow here from the properties of bound matter.

3. *The Experimental Facts*

3.1. *The Klemperer Experiment* (Klemperer, 1934)

Two Geiger counters are arranged geometrically with their windows parallel and facing each other; between them is placed a thin sheet of metal that contains a source of positrons. The metal sheet is thick

† The conventional interpretation of black-body radiation and its spectral distribution involves a photon gas that is supposedly uncoupled from charged matter. Within the framework of the study discussed here, these data can also be explained in terms of an ideal gas of particle–antiparticle pairs in their ground states of null energy-momentum (Sachs, 1965). This is the state that corresponds to the mathematical solution discussed in this article.

enough to (effectively) stop all escaping positrons from reaching the counters, but thin enough to allow 'radiation' to pass. The counters are controlled electronically to detect coincidences. It was found that the number of coincidences was equal to the number of positrons that were emitted from the source. From the measured radiation absorption, each radiation count was determined to have an energy that was of order m (i.e. the order of 0·5 Mev).† This result, then, was interpreted in terms of the annihilation process

$$e^- + e^+ \rightarrow \gamma(+\mathbf{k}) + \gamma(-\mathbf{k})$$

where $\pm\mathbf{k}$ denote the oppositely directed, simultaneously produced 'radiation'.

It is clear that the existence of the photons $\gamma(\pm\mathbf{k})$ is only *inferred* from the observed response of the charged matter in the counters to the charged matter in the source. This author's approach leads to predictions of this experimental result from an exact bound state solution of the coupled nonlinear field equations for the electron–positron pair. This solution [equation (3.1.4)] exhibits a dynamical motion in which the source terms in Maxwell's equations are a pair of mutually orthogonal currents that interact with external charges (the apparatus) from a common plane. Further, the response of the two Geiger counters would entail a transfer of energy from the pair that must equal the difference between the combined energy of the electron and positron when they are almost free (as they approach each other) and the energy that is associated with the bound state into which they will go. The former energy is $2m$. The latter energy was calculated from the bound-state solution (discussed above) and was found to be zero.‡ Thus, the prediction for the total amount of energy that is transferred to the Geiger counters is $2m$, thereby agreeing with the experimental facts.

To complete the comparison with the Klemperer experiment, it must be shown that this theory predicts that the charged matter in the *two counters will respond simultaneously* to the process in which the particle and antiparticle go into their *ground state* of null energy-momentum. This result follows automatically from the feature of

† Units are chosen with $\hbar = c = 1$.

‡ The conserved energy-momentum follows from the invariance of the Lagrangian formalism with respect to space–time translations. With the solution discussed above, this was computed to be a null-vector (see footnote on p. 394). It is also to be noted that a 'kinetic' term appears in both the initial and the final states. Thus, this term makes no contribution to the *energy transfer* that is sought and need not be introduced into the formalism.

this theory that one does not have two separate particles, defined in their own space–time coordinate systems. Rather, the fields that describe the interacting components of the system are mapped onto one space–time. The points in this coordinate space serve to localize the interactions between all of the particles that make up the system (in this case, the electron and the positron). Thus, the interactions to which both fields refer involve only *one time parameter*.

To be more explicit, consider the actual solutions that were obtained for this problem.† They follow from the uncoupled field equations

$$\begin{aligned} \{\gamma_\mu \partial_\mu - e^- \mathscr{I}(e^+) + m\} \psi^{(e^-)}(x) = 0 \\ \{\gamma_\mu \partial_\mu - e^+ \mathscr{I}(e^-) + m\} \psi^{(e^+)}(x) = 0 \end{aligned} \tag{3.1.1}$$

where $\mathscr{I}(e^\pm)$ is the field-coupling term (its explicit form is given in the footnote below) and the structure of the positron solution is defined in terms of that of the electron solution in the usual way, i.e.,

$$\psi^{(e^+)} = \gamma_2 (\psi^{(e^-)})^* \tag{3.1.2}$$

An important feature of the field equations (3.1) is the absence (from the outset) of any self-energy terms.

The bound-state solution of the field equations (3.1) that was found corresponds to the situation in which the electron and the positron are in the same state of motion, i.e., the source terms (aside from polarity) in the respective sets of Maxwell's equations for the particle and the antiparticle are the same. The latter equations were expressed in *spinor form* as follows (Sachs, 1964; Sachs & Schwebel, 1962):

$$\sigma_\mu \partial_\mu \varphi_\alpha^{(p)}(x) = p \bar{\psi}^{(p)}(x) \Gamma_\alpha \psi^{(p)}(x) \tag{3.1.3a}$$

where

$$\begin{aligned} \bar{\psi}^{(p)} \Gamma_1 \psi^{(p)} \equiv 4\pi i \begin{pmatrix} \bar{\psi}^{(p)}(-\gamma_0 + i\gamma_3)\psi^{(p)} \\ \bar{\psi}^{(p)}(i\gamma_1 - \gamma_2)\psi^{(p)} \end{pmatrix} \\ \bar{\psi}^{(p)} \Gamma_2 \psi^{(p)} \equiv 4\pi i \begin{pmatrix} \bar{\psi}^{(p)}(-i\gamma_1 - \gamma_2)\psi^{(p)} \\ \bar{\psi}^{(p)}(\gamma_0 + i\gamma_3)\psi^{(p)} \end{pmatrix} \end{aligned} \tag{3.1.3b}$$

† The parts of the theory dealing with electrodynamics are in Sachs, M. (1963). *Nuovo cimento*, **27**, 1138; **37**, 977 (1965); Sachs, M. and Schwebel, S. L. (1961). *Nuovo cimento*, Suppl. **27**, 197; Sachs, M. and Schwebel, S. L. (1962). *Journal of Mathematics and Physics*, **3**, 843; Sachs, M. and Schwebel, S. L. (1963). *Nuclear Physics*, **43**, 204; Hofstadter, R. and Schiff, L. I. (eds.) (1964). *Nuclear Structure* p. 336. Stanford.

where $p = e^+$ and e^-, $e^+ = -e^- = e$, and $\alpha = 1$, 2 represent the two independent (uncoupled) spinor field sources that appear in the electromagnetic equations.

With the positron and electron in the same state of motion, i.e.

$$\bar{\psi}^{(e^+)}(x)\,\Gamma_\alpha\,\psi^{(e^+)}(x) = \bar{\psi}^{(e^-)}(x)\,\Gamma_\alpha\,\psi^{(e^-)}$$

an examination of the coupled nonlinear field equations (3.1.1) led to the *exact solution*,

$$\psi^{(e^+)} = -\psi^{(e^-)} = \begin{pmatrix} \exp(-imt) \\ 0 \\ 0 \\ \exp(imt) \end{pmatrix} \tag{3.1.4}$$

The rigorous derivation of this solution is given in Section 6 and the appendix of Sachs and Schwebel (1961).

Two important features of the solution (3.1.4) are: (1) that it is independent of spatial coordinates, and (2) there is only one time parameter t in both of the field solutions $\psi^{(e^+)}$ and $\psi^{(e^-)}$. The two currents that are detected by each of the counters in this experiment are not the sources that correspond to the individual electron and positron. Rather, they are the independent spinor source fields, $e\bar{\psi}\Gamma_1\psi$ and $e\bar{\psi}\Gamma_2\psi$ that occur in the uncoupled two-component spinor formulation of electromagnetic theory (Sachs, 1964; Sachs & Schwebel, 1962). Thus, this feature would not have occurred had the usual vector representation of the Maxwell theory been maintained. The actual numbers, which are determined in the measurements discussed in this article, depend on the magnitude of the electromagnetic coupling constant e^2 and on the temporal behavior of the source fields. It should be emphasized that the source fields $\bar{\psi}\Gamma_1\psi$ and $\bar{\psi}\Gamma_2\psi$ are not identified here with 'free currents'; they rather have meaning only as *factors* which appear in combination with other spinor variables to form the expressions that represent the actual interactions between the particle–antiparticle pair and the apparatus. The coupling of the e^- component to positively: (or negatively-) charged components of the detecting apparatus is indistinguishable from the coupling of the e^+ component to the negatively- (or positively-) charged components of the detector, when the pair is in the ground state, described by the solution (3.1.4). Nevertheless, it is the existence of two *independent source* terms within this formalism, that implies the existence of two *distinguishable interactions*.

With the field solution (3.1.4) in equation (3.1.3b), the source terms of the spinor electromagnetic equations (for both the particle and the antiparticle) are as follows:

$$\begin{aligned} \bar{\psi}\Gamma_1\psi &= -8\pi i\begin{pmatrix} 1 \\ -\exp(2imt) \end{pmatrix} \\ \bar{\psi}\Gamma_2\psi &= 8\pi i\begin{pmatrix} -\exp(-2imt) \\ 1 \end{pmatrix} \end{aligned} \tag{3.1.5}$$

Thus, the actual electromagnetic forces that are deduced from the measurements of the detection process involve the coupling of these terms to the spinor field variables φ_α of the detecting apparatus (i.e. the Geiger counters). It has been shown in earlier publications (Sachs, 1964; Sachs & Schwebel, 1962) that there is a one-to-one correspondence between some of the force density terms in the spinor formulation

$$\varphi_\alpha^\dagger Y_\beta \qquad (\alpha, \beta = 1, 2) \tag{3.1.6}$$

where

$$Y_\beta \propto \bar{\psi}\Gamma_\beta\psi$$

and all of the components of the Lorentz force density in the standard vector language. (This correspondence has been shown to be the result of a topological feature in general relativity, as well as in special relativity.)

With the establishment of such a correspondence, it is perhaps more instructive for the purposes of this article, to express the complex source terms (3.1.5) in terms of the charge and current densities of the usual vector formalism. The identification is as follows†

$$\begin{aligned} e^\pm \bar{\psi}\Gamma_1\psi &= 4\pi i\begin{pmatrix} -\rho + j_3 \\ j_1 + ij_2 \end{pmatrix} \\ e^\pm \bar{\psi}\Gamma_2\psi &= 4\pi i\begin{pmatrix} -(j_1 - ij_2) \\ \rho + j_3 \end{pmatrix} \end{aligned} \tag{3.1.7}$$

† It should be noted that under Lorentz transformations, the functions on the left-hand side of equation (3.1.7) transform as spinors—they are not form invariant with respect to the vector transformations of the components ρ and j of a four-vector. However, the invariants in equation (3.1.6) (which entail the measuring apparatus through the spinor field variables φ_α) do indeed preserve the numerical mapping of the actual observations which relate to the Lorentz force density. The latter terms are topological invariants in general relativity and reduce to Lorentz invariants in special relativity (Sachs, 1964; Sachs & Schwebel, 1962).

Comparing equations (3.1.5) and (3.1.7), the source terms (for $\alpha = 1$ and 2) can be expressed with the usual variables as follows (in a unit volume):

$$\alpha = 1: \qquad \rho = 2e^{\pm}, \quad j_3 = 0, \quad j_1 + ij_2 = 2e^{\pm} \exp(2imt) \tag{3.1.8a}$$

$$\alpha = 2: \qquad \rho = 2e^{\pm}, \quad j_3 = 0, \quad j_1 - ij_2 = 2e^{\pm} \exp(-2imt) \tag{3.1.8b}$$

It follows from equations (3.1.5) [or from equations (3.1.8a) and (3.1.8b)] that the oppositely polarized currents are mutually transverse with respect to the x_3-direction and that the coincident response of the two counters would be to two spatially transverse currents that are 90° out of phase with each other. This assertion will be proven below. Before proceeding, however, it should be noted here that the two currents described above are *distinguishable* and correlated in the sense that two detectors on either side of the x_1–x_2 plane should detect them individually. Of course, since the two currents are linearly independent, one can always re-express them in terms of two linearly polarized, rather than circularly polarized currents. This is done by taking the sum and difference of equations (3.1.8a) and (3.1.8b) to give

$$j_1 = 2e^{\pm} \cos(2mt) \tag{3.1.8a$'$}$$

$$j_2 = 2e^{\pm} \sin(2mt) = 2e^{\pm} \cos(2mt - \pi/2) \tag{3.1.8b$'$}$$

The description in terms of circularly polarized currents (3.1.8a) and (3.1.8b) or linearly polarized currents (3.1.8a′) and (3.1.8b′) depends, of course, on the type of detecting apparatus that is involved in the experiment to be compared with theory.

It will now be shown from the form (3.1.8) of the linearly independent current terms for the particle–antiparticle pair, that the two detecting counters placed along a common axis that is perpendicular to the plane of a thin positron source would detect the oppositely polarized currents simultaneously, in agreement with the experimental result. To prove this assertion it will be necessary to calculate the electric field intensities $E_{\pm}$ (at the location of the detecting counters) that correspond to the current densities

$$j_{\pm} = j_1 \pm ij_2$$

at the electron–positron source. This, of course, is because it is the electric field intensity that determines the motion of a test charge in the detecting apparatus.

In the Lorentz frame of the detecting apparatus, the vector potential that corresponds to the polarized current densities $j_{\pm}$ is determined from the particular solutions of D'Alembert's equation

$$A_{\pm}(\mathbf{r}',t') = 4\pi j_{\pm}(\mathbf{r}',t') \tag{3.1.9}$$

where $(\mathbf{r}',t')$ are the coordinates of the test charge in the apparatus, while $(\mathbf{r}=0,t)$ are the coordinates in the Lorentz frame of the pair itself. The solution of equation (3.1.9) is

$$A_{\pm}(\mathbf{r}',t') = \int j_{\pm}(t)S(x-x')\,d^4x \tag{3.1.10}$$

where $S(x-x')$ is the Green's function for D'Alembert's equation and x stands for the world point $(\mathbf{r},t)$. It is well-known that the form of $S(x-x')$ is not unique—i.e. if we insist that $t>0$ then only the retarded form appears, while with $-\infty<t<\infty$ both the retarded and advanced forms can be used. This author's theory requires that the fundamental formalism must be symmetric with respect to the interchange of the 'emitter' and the 'absorber' field variables in the description of the elementary interaction. Thus, it is required here to use the Green's function that is an average of the retarded and advanced forms. This has the following form in terms of the Dirac delta functions:

$$S(x-x') = \frac{1}{2|r-r'|}\{\delta[(t-t')-|\mathbf{r}-\mathbf{r}'|]+\delta[(t-t')+|\mathbf{r}-\mathbf{r}'|]\} \tag{3.1.11}$$

It should be noted at this point that the result to be derived below, which shows that two counters on opposite sides of an axis perpendicular to the plane of polarization of the currents $j_{\pm}$ would respond simultaneously to the correlated currents, is insensitive to the appearance or lack of appearance of the advanced solution. The same result would be obtained should we use only the retarded potential, corresponding to the Green's function

$$S(x-x') = \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta[(t-t')+|\mathbf{r}-\mathbf{r}'|]$$

which is adopted conventionally *in the particle theories* because of the requirement of causality.

Since $j_3=0$, the substitution of equation (3.1.11) into equation (3.1.10) (with $\mathbf{r}=0$) gives the following form for the vector potential:

$$A_3(r',t') = \int j_3(t)S(x-x')\,d^4x = 0$$

$$A_{\pm}(\mathbf{r}',t') = \frac{1}{2}\frac{(2e\pm)}{r'}\{\exp[\pm 2im(t'+r')]+\exp[\pm 2im(t'-r']\}\,\hat{\mathbf{e}}_{\pm} \tag{3.1.12}$$

where $\hat{\mathbf{e}}_{\pm} = \hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_i$ are the unit vectors in the ith direction. It then follows from equation (3.1.12) that the electric field intensities $E_{\pm}(\mathbf{r}', t')$ at the sites of the counters, representing the current densities $j_{\pm}(t)$ at the particle–antiparticle source, have the form

$$\begin{aligned} E_{\pm}(r', t') &= -\frac{\partial A_{\pm}}{\partial t'} \\ &= \frac{\pm(2ime^{\pm})}{r'}\{\exp[\pm 2im(t'+r')] + \exp[\pm 2im(t'-r')]\} e_{\pm} \end{aligned} \tag{3.1.13}$$

Thus we see that $E_{+}(r', t')$ describes a wave motion for an oscillating charge with frequency $\omega = 2mc^2/\hbar$ and a propagation vector whose magnitude is ω/c. (The constants $\hbar$ and c are inserted here only for illustrative purposes. Recall that we have been using units throughout this analysis with $\hbar = c = 1$.)

We see, then, that when the phase of the current source is fixed, say at zero, corresponding to a specific time ($t = 0$), then the phase of the corresponding electric field vector at the detecting apparatus does not become zero until $t' = r'/c$ in the retarded solution and $t' = -r'/c$ in the advanced solution. Thus, the *magnitude* of the time for propagation of the interaction between the pair and each of the detectors which are located at a distance r' from the pair is r'/c. The salient point to be noted at this stage is that for the solution E_{+}, the sign of the propagation vector $\mathbf{k}$ is positive in the retarded term and negative in the advanced term. The oppositely polarized current j_{-} gives rise to the electric field intensity E_{-} with the same form as E_{+} except that the propagation vector in this case is negative in the retarded term and positive in the advanced term. Thus we see that the oppositely polarized currents $j_{\pm}$ at the common spatial location ($\mathbf{r} = 0$) give rise to oppositely polarized electric field vectors accompanied by opposite directions of propagation of interaction with the apparatus, and when each counter is an equal distance r' from the pair, along a common axis with the source at the center, they will *simultaneously* detect oppositely polarized currents at the time $|t'| = |r'/c|$.

A further comment should be made here in regard to the role of the advanced solution in this theory. The retarded solution implies that a signal was emitted at the earlier time $t < t'$, while the advanced solution implies that the signal was emitted at the later time $t > t'$. Of course, if one should insist on viewing the interaction in terms of a model where a 'thing' (the emitter) ejects another 'thing' (the signal) and that the signal is then absorbed by a third 'thing' (the absorber) then in view of the common sense notion of causality in the strict sense of cause-effect relation, it would only make logical sense to use the retarded solution. On the other hand, the theory discussed here is not a particle theory. The fundamental entity is the 'elementary interaction'—where the 'emitter', 'absorber' and 'signal' are not separate 'things' with their own space–time trajectories. Instead there is in this *field theory* only one space–time coordinate system which in turn is used to facilitate a mapping of the fundamental field variables. Thus, there is no reason here to reject the advanced solution. Indeed, the latter is required according to the elementary interaction field theory in order to obtain a symmetry (in the Lagrangian description) between the emitter and absorber components of *the single elementary interaction*. The same symmetry was necessarily imposed in the Wheeler–Feynman action-at-a-distance approach (Wheeler & Feynman, 1945) but not entirely for the same reasons. A basic difference between the elementary interaction field theory and the approach of Wheeler and Feynman lies in the field versus particle description. In this author's approach, one constructs a single system with many coupled field components, each mapped in a common space–time. On the other hand, the Wheeler–Feynman theory considers many independent particles, each in terms of its own space–time trajectory, symmetrically emitting to and absorbing from the companion particles that make up the system. Thus, the elementary field theory considers n coupled fields in a 4-dimensional space, while the Wheeler–Feynman theory considers a $4n$-dimensional space for the n-particle system. In both theories, however, use must be made of the advanced solutions of Maxwell's equations by introducing them on an equal par with the retarded solutions. But as we have noted earlier, had we used the retarded solutions alone, rather than an average of the advanced and retarded solutions, the present derivation still predicts that the oppositely polarized current sources, j_+ and j_-, respectively give rise to interaction propagation along the x_3 axis in opposite directions and with a common speed c. It then follows that the information given to the counters along a common axis on either side and equidistant from a thin positron source that is perpendicular to this axis, should arrive *simultaneously*, each with energy equal to mc^2, as it was observed in the Klemperer experiment.

The latter result pertaining to energy follows from the insertion of the solutions (3.1.4) in the Lagrangian density. Using Noether's theorem, the conserved energy-momentum for this state can then be computed. It was found† to be a null-vector, with each component

† See footnote on page 394.

itself being zero. Consequently, the quantity of energy that is transferred from a given electron–positron pair to the counters is equal to that which is required to totally uncouple this tightly bound pair. Since the energy of the pair when they are in the asymptotic state corresponding to 'free particles' is equal to $2mc^2$, the transferred energy is just equal to this amount. Since this total energy transfer from the pair is equally divided between the two counters in the experiment described, each of the counters must absorb the quantity of energy equal to mc^2.

It is concluded, then, that the present theory predicts the observed results of the Klemperer experiment. Of course, the results relating to energy–momentum were in any case anticipated from both the present theory and from the conventional particle theory on the grounds of energy and linear momentum conservation. The difference in the assertions of the two theories, however, lies in the *deterministic field approach* of this derivation as compared with the *intrinsically statistical approach* of the usual postulation regarding pair annihilation.

3.2. *The Wu-Shaknov Experiment.* Wu & Shaknov (1950)

According to the conservation of angular momentum, it follows that the absence of angular momentum in the interacting electron–positron system (just prior to annihilation) implies that the two photons which are created must be oppositely polarized in a mutual plane that is transverse to their oppositely directed motion. Further the quantum mechanical requirement for an antisymmetric wave function to describe the pair in the singlet state implies that the final state of two photons must correlate their polarization vectors with a 90° difference in phase angles. To test the latter consequence of the quantum theory, Wu and Shaknov designed an experiment to measure the coincident counts of two linearly polarized radiation fields that have been scattered through an angle θ, relative to their initial direction of propagation. The significance of this experiment lies in the sensitivity of the cross section for the scattering of coincident radiation beams to the correlation of their intrinsic polarizations.

When the mechanism of scattering is the Compton effect and when the two radiation fields are perpendicularly polarized, it follows from the Klein-Nishina formula that the ratio of coincident counts for perpendicularly linearly polarized radiation is exactly 2·00 for cases when the scattering planes (formed by the initial and scattered directions of radiation) are perpendicular and parallel and where the angle of scattering in these planes is averaged over. The ratio that was measured by Wu and Shaknov was $2{\cdot}04 \pm 0{\cdot}08$. Consequently, the experiment confirmed the quantum mechanical prediction of a 90° phase correlation in the coincidentally scattered radiation beams. This result, then, ruled out the hypothesis of Furry (1936) which suggested that the noninteracting partial systems should become uncorrelated.

The Wu-Shaknov result is also a confirmation of the prediction of the nonlinear field theory that is discussed in this paper. While there is no free radiation involved here, the two independent source fields [equations (3.1.5) or equations (3.1.8a) and (3.1.8b)], of the spinor formulation of electromagnetism, are correlated with polarizations that are 90° out of phase. Thus, a detecting apparatus that is sensitive to linear polarization would detect two different currents described in equations (3.1.8a) and (3.1.8b). Note once again that these currents are defined at a common time t. Thus, the polarizations are correlated as the experiment implies.

3.3. *The Compton Effect*

Since the conclusions from the results of the Wu-Shaknov experiment depend on an identification with the conventional expression for the Compton scattering cross-section, it becomes necessary here to show how the same expression appears with the present nonlinear field theory. When one considers the process in which the electron–positron pair is scattered by an electron, and when the assumption is made that the 'projectile' and the 'target' are sufficiently weakly coupled, then the formalism's three coupled field equations (for the coupled electron–positron pair and the target electron) effectively uncouple into the separate coupled field equations for the pair alone, and the field equation for the target electron. In this approximation, the latter field equation approaches the standard linear Dirac equation for a free particle. The next step (in deriving the Klein-Nishina formula) is to introduce a small coupling between the pair and the target electron and to treat this as a perturbation on the free particle solutions for the target electron. However, we have seen that when the electron–positron pair is in its ground state (of null-energy momentum), it behaves dynamically as a pair of oppositely circularly polarized currents (or equivalently, as a pair of perpendicularly linearly polarized currents) whose phases are correlated with a 90° difference and whose plane of polarization is perpendicular to the axis along which the interaction with other charged matter is propagated. Thus, the pair in this state has the same formal dynamical properties as the two photons that are conventionally described in the pair annihilation process.† Further, the electromagnetic potential which corresponds to the independent currents that were derived [equations (3.1.8a) and (3.1.8b)] has the same time behavior, according to Maxwell's equations for this system. If one now makes a Lorentz transformation to the rest frame of the (assumed uncoupled) target electron, rather than to a stationary apparatus, then the effective vector potential that acts on the latter takes the following form (in a unit volume)

$$A_3 = 0, \qquad A_{\perp} \propto e^{\pm} \exp[\pm i(\omega t - \mathbf{k}.\mathbf{r})] \tag{3.3.9}$$

where

$$\omega = 2m[(1-v)/(1+v)]^{1/2}$$

and v is the relative velocity between the target and the projectile systems.

Thus, with the exact solution (3.1.4) for the ground state of the particle–antiparticle pair, and the approximation of weak coupling between the pair and the electron target, the usual formal perturbation technique can be applied to determine the cross-section for scattering. The perturbing interaction is $e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$, where ψ is the free field spinor solution for the target electron and A_{μ} is the effective electromagnetic potential for the pair [equation (3.1.12)]. For the solution considered, the coulomb potential A_0 for the electron–positron pair is zero. This is demonstrated in the longitudinal part of the coupling term since with the solution (3.1.4) both the e^+ and the e^- components have the same dynamics and they are each acting on other charged matter from a common point in space with effective charge that is equal in magnitude, but oppositely polarized.

The interaction $e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ is isomorphic with the usual expression for the coupling of a free electron to a free radiation field, when the solutions (3.1.12) are used for the electron–positron pair. It then follows that the Klein–Nishina formula for the Compton cross-section follows in the usual way (Heitler, 1944). Thus it is concluded that the predictions of this theory are identical with those of the usual quantum mechanical approach in regard to the correlated polarizations of distinguishable currents that were measured in the Wu–Shaknov experiment. It should be noted, however, that if the coupling energy between the pair and the scattering charge should become increasingly

† One may *not* call this bound state 'photons' because (a) the positron and electron in this state have inertial properties, and (b) they each couple electromagnetically to other charged particles, i.e. a nonzero source term exists in Maxwell's equations for the individual electron and positron that comprise the bound pair.

strong, the tendency would be to excite the pair into a state that no longer displays the dynamical properties of 'photons', but rather indicates a behavior that would be interpreted as 'pair creation'. To describe the latter in a quantitative fashion will require the consideration of the solutions of three coupled nonlinear field equations—for the projectile pair and the target electron, all strongly coupled.

4. *Conclusions*

In summary, the exact bound-state solution (3.1.4) of the coupled nonlinear field equations for the particle–antiparticle pair, leads to physical predictions that entirely agree with the experimental observations that are normally attributed to pair annihilation. Yet, matter is not annihilated nor do photons play any role in the description. Further, the correlation between separate things that seems to be implied by the experimental data (and strengthens the argumentation which leads to the EPR paradox) follows here from a description in terms of *one field*, where things are not, in fact, separated at all. These results, which are sensitive to an exact solution of a deterministic field theory, are then a test of an approach in which the EPR paradox (an example of a Class One difficulty) does not appear because of the completeness of the underlying description of the *elementary interaction*.

It is interesting to note the contrast between the approach discussed here and that of a recent phenomenological theory of Schwinger (1966, 1967). According to the latter theory, the propagators in the formalism of quantum electrodynamics should be interpreted in terms of the continuous creation and annihilation of arbitrary numbers of particles, antiparticles and photons. The 'sources' of the created matter and radiation are the presence in space–time of interacting particles of matter themselves—just as the presence in space and time of charged particles, in the conventional interpretation of quantum electrodynamics, implies the creation and annihilation of photons through the emission and absorption of energy and momentum by the interacting particles of matter. Schwinger's interpretation presents a different way of looking at the formalism of quantum electrodynamics that is perhaps more reasonable than the conventional approach. The approach discussed in this paper differs from Schwinger's in that (1) particles and antiparticles are not annihilated or created from a vacuum, and (2) it entails a model which requires an entirely different mathematical description—one that would be incompatible with the formalism of quantum field theory. This is a nonlinear, 'classical' spinor field theory, where quantization plays no role and where 'photons' do not appear. Further, there is no concern here to provide a 'mechanism' for the annihilation and creation of matter. In this approach, the interacting particles are assumed to exist from the outset—the only 'mechanism' that is needed is the one to explain the bound states of the electron–positron system. This is spelled out explicitly in the field equations.

Finally, it should be noted that there is basic agreement between the elementary interaction approach and the attempt of the quantum theory to incorporate the act of measurement into the mathematical description. Yet, in contrast to the quantum theory, the present approach's incorporation of the measurement is a complete one in the sense of providing, at the fundamental level, a closed and dynamically complete accounting of all variables, to arbitrary accuracy, for the entire system of 'observed' and 'observer'. Thus, within this approach, the statistical aspects of the linearized equations (that follow as a limit of the exact nonlinear formalism) are *subjective.* This is because they entail a lack of information due to an insufficient amount of energy-momentum transfer between the different parts of the system. In the problem under study, particular properties of the pair itself are *completely determined* by the corresponding solution given in equation (3.1.4). The lack of complete information in a macroscopic measurement reside in the description of the coupling of this pair to a macroscopic measuring apparatus. Nevertheless, in the limit in which there would be sufficiently high energy-momentum transfer between the pair and the apparatus, complete information about the system would again emerge according to the solutions of the field equations and the statistical features of the description would vanish. Thus, the *underlying theory* relates to a closed system—it is therefore entirely *objective*. These points are discussed further by Sachs (see footnote on p. 388.)

In practice, the limit of complete knowledge cannot be reached. This is because the requirement of scientific reproducibility of data would always require an 'apparatus' to 'stand aside' and perceive the remainder of the system, without appreciably affecting the spectrum of dynamical properties of the latter. Nevertheless, the way in which the present approach must arrive at the predictions for this apparatus is to start with the complete formalism for the whole system and then to take the limit in which that portion of the system that can be called 'apparatus' becomes uncoupled from the rest of the (otherwise coupled) system. This is exemplified in the present study by that part of the system that is identified with the *strongly coupled pair*. The results depended on the fact that the crucial solution followed from nonlinear field equations—there would have been no way of arriving at the results from a linearized version of the field equations for the pair.† Thus, within the present conceptual approach, one arrives at a nonlinear and closed structure for the fundamental field equations and a uniqueness in the limits of their solutions. It is precisely for this reason that the Einstein-Podolsky-Rosen paradox is removed by the present formulation.

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† The situation usually treated (to exemplify the EPR paradox) of a compound system of two atoms that were previously bound in a known spin state, does not lead to as strong an argument as does the example of 'pair annihilation'. This is because the breakup of a molecule into its atomic components would lead, within the present formalism, to a linearized set of uncoupled equations for the separated atoms, that are the same as the standard equations of nonrelativistic quantum mechanics. The correlation in their wave functions persists, within this approach, because the latter is only *an approximation* for the correlated system that is *never actually uncoupled*. (This result is in agreement with Bohr's contention about the nonseparability of the partial systems.) However, the example of 'pair annihilation' does not entail any ambiguity since it is not based on any approximation. The predictions here, that relate to the experimental facts, are a consequence of *an exact solution* of deterministic field equations.

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